Problem Set 6 Solutions
CS&SS Math Camp 2020

1. Let Y be a uniform random variable on the interval $[2,10]$.

   (a) Compute the expected value $(E[Y])$.

   $$E[X] = \int_2^{10} x \cdot \frac{1}{10-2} dx = \frac{1}{8} \int_2^{10} x dx = \frac{1}{8} [x^2/2]_2^{10} = \frac{1}{8} [100/2 - 4/2] = 6$$

   (b) Compute the variance $(Var[Y])$.

   $$V[X] = \frac{1}{8} \int_2^{10} (x-6)^2 dx = \frac{1}{8} \cdot \frac{1}{3} [(x-6)^3]_2^{10} = \frac{1}{24} [(10-6)^3 - (2-6)^3] = 5.33\bar{3}$$

2. A family has 4 pets, let X denote the number of cats. Assume the only pets are cats or dogs and they are assigned to families in equal probability.

   (a) Write down the probability distribution of X. Hint: start by writing down the sample space and count the number of ways each event could occur. Slide 10 from Lecture 6 should be helpful.

X-Value	0	1	2	3	4
$P(X = x_i)$	1/16	4/16	6/16	4/16	1/16

   (b) Compute the expected value $(E[X])$.

   $$E[X] = 0(1/16) + 1(4/16) + 2(6/16) + 3(4/16) + 4(1/16) = 2$$

   (c) Compute the variance $(Var[X])$.

   $$V[X] = (0-2)^2(1/16)+(1-2)^2(4/16)+(2-2)^2(6/16)+(3-2)^2(4/16)+(4-2)^2(1/16) = 1$$

3. Toss a coin 4 times, let X denote the number of heads.

   (a) Write down the probability distribution of X. Hint: start by writing down the sample space and count the number of ways each event could occur. Slide 10 from Lecture 6 should be helpful.

X-Value	0	1	2	3	4
$P(X = x_i)$	1/16	4/16	6/16	4/16	1/16

    (b) Compute the expected value $(E[X])$.

$$E[X] = 0(1/16) + 1(4/16) + 2(6/16) + 3(4/16) + 4(1/16) = 2$$

    (c) Compute the variance $(Var[X])$.

$$V[X] = (0-2)^2(1/16) + (1-2)^2(4/16) + (2-2)^2(6/16) + (3-2)^2(4/16) + (4-2)^2(1/16) = 1$$

4. John pays \$40 per year for towing insurance. He thinks the probability that he will need to have his car towed is 10% and the probability that he will need to have it towed more than once is zero. Without insurance the cost of towing is \$100, but the cost is zero if insured. Let X=John's expenses next year for towing and/or insurance.

    (a) If he buys insurance, what is the value of X?

$$X = \$40$$

    (b) If he doesn't buy insurance, what two values can X take?

$$X = \$0 \text{ or } \$100$$

    (c) Find $E[X]$ for both (a) and (b). Should he buy the insurance?
    Buys insurance: $E[X] = 1 \cdot 40 = \$40$
    Does not buy insurance: $E[X] = 0.1 \cdot 100 + 0.9 \cdot 0 = \$10$
    He expects to pay less if he doesn't buy insurance, so he should not buy it.

5. Let X represent the number of jobs held during the past year for students at a school, and suppose X has the following probability distribution:

X-Value	0	1	2	3	4
$P(X = x_i)$	0.15	0.28	0.36	0.10	0.11

    (a) What is the probability that a randomly selected student has fewer than two jobs?

$$P(X = 0) + P(X = 1) = 0.15 + 0.28 = 0.43$$

    (b) Find $P(X > 0)$.

$$P(X > 0) = 1 - P(X = 0) = 1 - 0.15 = 0.85$$

    (c) Find $P(X > 2)$.

$$P(X > 2) = P(X = 3) + P(X = 4) = 0.10 + 0.11 = 0.21$$