# Problem Set 5: Probability
# Math Camp 2020

1. Suppose we have three groups.
   A = { Warren, Sarah Beth, Shane, Leah, Seth }
   B = { Michael, Elizabeth, Seth, Shane, David }
   C = { Emily, Megan, Ian, Leah }

   $A \cup B =$ { Warren, Sarah Beth, Shane, Leah, Seth, Michael, Elizabeth, David }

   $A \cap B =$ { Shane, Seth }

   $B \cap C = \emptyset$

   What do we call events B and C? mutually exclusive

2. Suppose there is an urn with 1 blue, 1 green, 1 red, and 1 purple ball. I draw two out randomly. What is the sample space of this experiment? (i.e. what are the possible outcomes)

   S = { BG, BR, BP, GR, GP, RP }

   Let R be the event that one of yours balls was red. $P(R) = \frac{3}{6}$

   Let B be the event that one of yours balls was blue. $P(B) = \frac{3}{6}$

   $P(B^C) = \frac{3}{6}$

   $P(R \cap B) = \frac{1}{6}$

   $P(R \cup B) = \frac{5}{6}$

   $P(R|B) = \frac{1}{3}$

   Are the events R and B independent? They are not independent since $P(R|B) \neq P(R)$.

3. Let $A$ be the event that a student passes her qualifying exams for graduate school on the first try and $B$ be the event that the student attends Math Camp. Suppose $P(A) = 0.90$ and $P(A|B) = 0.95$. Are $A$ and $B$ independent? What does this say about the effectiveness of Math Camp?

   Since $P(A|B) = 0.95 \neq 0.90 = P(A)$, events $A$ and $B$ are not independent.

4. The probability of infecting another household member with a certain strain of influenza during one day is 0.10. Suppose that in a household of four people, three of them are sick.

   (a) What is the probability that the fourth person becomes infected during a single day of exposure to the three sick household members?

   It is much harder to count all the ways the fourth person becomes infected (person 1 infects, person 2 infects, persons 2 and 3 infect, etc.) than the number of ways the fourth person does not become infected. The probability the fourth member escapes infection during a single day of exposure is equal to the probability person 1 does not infect them times person 2 does not infect them times the probability person 3 does not infect them, or

$$(1 - .10) \times (1 - .10) \times (1 - .10) = (1 - .10)^3.$$

   So the probability the fourth member becomes infected during a single day is:

$$1 - (1 - 0.10)^3 = 1 - 0.9^3 = 1 - 0.729 = 0.271$$

   (b) What is the probability that the fourth person becomes infected during two days of exposure to the sick household members?

   Again we will use the complement. Using the answer from part (a): the probability of escaping infection for two consecutive days is: $(1 - 0.271)^2 = 0.729^2$, so the probability of becoming infected during two days of exposure is

$$1 - 0.729^2 = 1 - 0.53 = 0.47$$