Problem Set 3: Differential Calculus
CS&SS Math Camp 2020

1. Plot the function $f(x) = 3x + 2$.

   (a) By eye, what is the derivate of $f(x)$ at $x = 4$, i.e. what is the slope of the tangent line to the curve at $x = 4$?

   (b) Compute the derivative using the appropriate formula.

   Compute the derivative:

2. $f(x) = x^5$

3. $f(x) = 10x - 30$

4. $f(x) = 2x^4 + x^2$

5. $f(x) = tan(x)$

6. $f(x) = e^{sin(x)}$

7. $f(x) = xe^x + log(sin(x))$

We can also have a function of a different variable besides x. This is just changing the variable name and you will see this a lot in your statistics methods classes.

8. Compute the derivative of $g(\theta) = \theta^2 - \theta^4$

9. Find the global minimum of $f(z) = z^2 - 6z + 8$

10. In the following function, treat x as a constant, and differentiate with respect to μ:
$h(\mu) = x\mu^2$

Bonus In the following function of μ, treat $X_1, X_2, ..., X_n$ (your data) as constants. Maximize the function over μ (the population mean). In other words, find the value of μ, expressed in terms of $X_1, X_2, ..., X_n$, at which the function reaches its global maximum. HINT: review your rules of logarithms and exponents to simplify the expression before taking the derivative.
$L(\mu) = log(\frac{1}{\sqrt{2\pi}}e^{-\sum_{i=1}^{n}(X_i-\mu)^2})$